The "Usual" Dilemma

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1 Problem

We've all wondered about this, right? When you go to a restaurant multiple times, should you try something new or just order your "usual"? Well, wonder no more! Thanks to the wonders of mathematics, we are now in a position to definitively answer this long-standing conundrum![*]

1.1 Assumptions

Assume that you're going to eat at a restaurant t times. The menu has m items on it.

Assume that you can rank your preferences for each of the m items with a different score of 1,2,3,..., or m. A higher score indicates a better meal.

Assume that you have no idea how an item will taste/score before ordering it.[†]

Assume that after eating an item, you only learn its relative ranking among the items you have eaten and not its absolute score.

Assume that your preferences don't change. For instance, you don't get tired of eating the same item over and over.

Assume you order exactly one item on each visit.

Assume that after trying v (for variety) items from the menu, you'll then order your favorite meal (the one with the highest score) for the rest of your $t-v$ visits.

Assume that the measure of your overall enjoyment of the t meals is expressed as the sum of the scores of each meal you ate.

[*] Under certain idealized assumptions.

[†] If it helps with credulity, assume you don't even know what the menu item is until you order it. i.e. they are simply listed as item 1, item 2, item 3, ... , item m, with no description.

1.2 Question

If you're going to visit a restaurant t times and there are m items on the menu, how many different items v should you try in order to maximize your expected enjoyment of all t meals?

You can think of expected enjoyment as if you were going to run the experiment many times of going to the restaurant t times and sampling v different meals randomly before settling on your favorite and eating your favorite $t - v$ times. What value of v will give you the best chance of yielding the most enjoyment?

1.3 Example

Let's say that you're going to visit a restaurant 3 times. The restaurant has 3 items on the menu. How many different items should you try?

If you tried one item, then there are three cases:

- $\{1, 1, 1\}$
- $\{2, 2, 2\}$
- $\{3, 3, 3\}$

Either you eat the worst meal 3 times, the second-best meal three times, or the best meal three times. So the three possible scores you could get are $1+1+1 = 3$, $2+2+2 = 6$, or $3+3+3 = 9$. Therefore the expected value of this strategy is $(3 + 6 + 9)/3 = 6$.

If you tried two items, then there are three cases:

- $\{1, 2, 2\}$
- $\{1, 3, 3\}$
- $\{2, 3, 3\}$

The three possible scores you could get are $1 + 2 + 2 = 5$, $1 + 3 + 3 = 7$, or $2 + 3 + 3 = 8$. Therefore the expected value of this strategy is $(5 + 7 + 8)/3 = 20/3 = 6\frac{2}{3}$.

If you tried three items, then there's one case:

- $\{1, 2, 3\}$

You simply try all of the items on the menu. So the value of this strategy is $1 + 2 + 3 = 6$.

Therefore if you're going to visit a restaurant 3 times with 3 items on the menu, you should only try 2 of the items and then eat your favorite of those 2 for your third visit in order to maximize your expected enjoyment.

2 Solution

2.1 Definitions

Let **f(m,t,v)** = The expected score of going to a restaurant with m items on the menu, t times, and trying v different items before settling on your favorite item for the remaining $t - v$ visits.

Let **g(m,v)** = The expected average score of the v different meals that you try from the m items on the menu.

Let **h(m,v)** = The expected score of the best meal you find when trying the v different meals from the m items on the menu.

Since the strategy is to eat v different meals and then eat the best of those v meals $t - v$ times, we can write the expected total score as

$$\boxed{f(m,t,v) = v \cdot g(m,v) + (t-v) \cdot h(m,v)} \tag{1}$$

2.2 Solving for g(m,v)

What's the average expected score of the v different meals that you try from the m items on the menu?

Let's look at $g(m,1)$. This is the expected vaue of one meal from the menu. There are m possibilities for the score: 1, 2, 3, 4, ..., m. Therefore, the expected value is

$$g(m,1) = \frac{1+2+3+...+m}{m} \tag{2}$$

It's a well known fact that the sum of the first m integers is

$$1+2+3+...+m = \frac{m(m+1)}{2} \tag{3}$$

You can see this by pairing up 1 and m, 2 and $m-1$, 3 and $m-2$, etc... These all sum up to $(m+1)$. There are $\frac{m}{2}$ such pairs.

Substituting (eq 3) into (eq 2) we find

$$g(m,1) = \frac{1+2+3+...+m}{m} = \frac{m(m+1)}{2m} \tag{4}$$

Simplifying gives

$$\boxed{g(m,1) = \frac{m+1}{2}} \tag{5}$$

Now let's look at $g(m,2)$. This is the expected average score of trying 2 different meals from the menu.

First, consider $g(4,2)$:

$$g(4,2) = \frac{(1+2)+(1+3)+(1+4)+(2+3)+(2+4)+(3+4)}{6 \cdot 2} \tag{6}$$

This is the expected average score of all of the possible combinations of 2 meals that you could eat from a menu of 4 items. There are 6 combinations and 2 meals in each combination. We divide by 6 to get the expected total score of each of the combinations. And we divide by 2 to calculate the average score of each individual meal.

You can see that there are 3 copies of each number in the sum in (eq 6). In general, each item can be paired with $m-1$ other items. So

$$g(m,2) = \frac{1 \cdot (m-1) + 2 \cdot (m-1) + 3 \cdot (m-1) + ... + m \cdot (m-1)}{\binom{m}{2} \cdot 2}$$

There are $\binom{m}{2}$ ways* to pick 2 items from m items. So we divide by $\binom{m}{2}$ to find the expected total score of all of the possible sums of different items you could pick. And we divide by 2 to find the average value of each individual meal since there are two meals in each combination.

Simplifying, we find

$$\begin{aligned}
g(m,2) &= \frac{(m-1)(1+2+3+...+m)}{\binom{m}{2} \cdot 2} \\
&= \frac{(m-1)}{\binom{m}{2} \cdot 2} \cdot \frac{m(m+1)}{2} \\
&= \frac{(m-1)}{\frac{m!}{2!(m-2)!} \cdot 2} \cdot \frac{m(m+1)}{2} \\
&= \frac{(m-1)(m-2)!}{m!} \cdot \frac{m(m+1)}{2} \\
&= \frac{(m-1)!}{m!} \cdot \frac{m(m+1)}{2} \\
&= \frac{1}{m} \cdot \frac{m(m+1)}{2} \\
&= \frac{m+1}{2}
\end{aligned}$$

*The choose function $\binom{n}{k}$ specifies how many different sets of k elements can made from a bigger set of n elements. $\binom{n}{k} = \frac{n(n-1)(n-2)...(n-k+1)}{k!} = \frac{n!}{k!(n-k)!}$
To see why this is true, consider picking the first element. There are n choices. For the second element there are $n-1$ choices. Keep picking until you've picked k elements. Since you're constructing a set, the order in which you picked the elements doesn't matter. There are $k!$ ways to represent a list of k elements. Therefore divide by $k!$ to remove the duplicates.

We can extend this reasoning to the general case of $g(m, v)$. For each item on the menu 1,2,3,...,m, we pick $v-1$ items to go with it since we're trying a total of v items. There are $\binom{m-1}{v-1}$ ways to pick $v-1$ items from $m-1$ items. We pick from $m-1$ items because we can't pick the same item twice.

There are $\binom{m}{v}$ ways to pick v items from a total of m. These are all of the ways that we can pick our v different meals. We divide by this value to get the expected total score. And since we're eating v meals, we divide by v to find the average expecred score of each individual meal. So we see that:

$$\boxed{g(m,v) = \frac{\sum_{i=1}^{m} \binom{m-1}{v-1} \cdot i}{\binom{m}{v} \cdot v}} \tag{7}$$

Simplifying we get

$$\begin{aligned}
g(m,v) &= \binom{m-1}{v-1} \frac{\sum_{i=1}^{m} i}{\binom{m}{v} \cdot v} \\
&= \frac{\binom{m-1}{v-1} \frac{m(m+1)}{2}}{\binom{m}{v} \cdot v} \\
&= \frac{\frac{(m-1)!}{(v-1)!((m-1)-(v-1))!} \cdot \frac{m(m+1)}{2}}{\frac{m!}{v!(m-v)!} \cdot v} \\
&= \frac{\frac{m!}{(v-1)!(m-v)!} \cdot \frac{m+1}{2}}{\frac{m!}{(v-1)!(m-v)!}} \\
&= \frac{m+1}{2}
\end{aligned}$$

So we see that in general

$$\boxed{g(m,v) = \frac{m+1}{2}} \tag{8}$$

In retrospect this is sort of obvious. The expected value of any random meal is $\frac{m+1}{2}$, so the average expected value of v meals will also be $\frac{m+1}{2}$. All this means is that if you're choosing the meals completely randomly, you can expect to, on average, get an average meal.

In order to improve the overall score of all of the meals, we need to skew the odds in our favor. To do that, we'll eat the best of the first v meals for all of the rest of the $t - v$ meals.

2.3 Investigating h(m,v)

What's the expected best score among v randomly selected meals?

We already know from our calculation of $g(m,v)$ that the expected score of 1 meal is $\frac{m+1}{2}$:

$$\boxed{h(m,1) = g(m,1) = \frac{m+1}{2}} \tag{9}$$

Now let's think about $h(m,2)$. This is the expected best score of two randomly picked meals.

Consider $h(4,2)$:

$$\begin{aligned} h(4,2) &= \frac{\max(1{,}2) + \max(1{,}3) + \max(1{,}4) + \max(2{,}3) + \max(2{,}4) + \max(3{,}4)}{6} \\ &= \frac{2+3+4+3+4+4}{6} \\ &= \frac{20}{6} \end{aligned}$$

Notice that $g(4,2) = \frac{4+1}{2} = \frac{5}{2}$. So even trying 2 meals compared to 1 improves the average expected meal from a score of 2.5 to 3.33.

Let's calculate $h(3,2)$:

$$\begin{aligned} h(3,2) &= \frac{\max(1{,}2) + \max(1{,}3) + \max(2{,}3)}{3} \\ &= \frac{2+3+3}{3} \\ &= \frac{8}{3} \end{aligned}$$

Let's see if we can find a pattern. We saw that $g(m,v) = \frac{m+1}{2}$, so maybe $h(m,v) = (m+1)$ times something.

$$h(3,2) = \frac{8}{3} = (m+1)x = 4x \Rightarrow \quad x = \frac{2}{3} \Rightarrow h(3,2) = (m+1)\cdot\frac{2}{3}$$

$$h(4,2) = \frac{20}{6} = (m+1)x = 5x \Rightarrow \quad x = \frac{2}{3} \Rightarrow h(4,2) = (m+1)\cdot\frac{2}{3}$$

So maybe in general $h(m,2) = (m+1)\cdot\frac{2}{3}$

But what is $h(m,v)$ for different values of v? Let's calculate some other examples:

$$
\begin{aligned}
h(2,1) &= \frac{1+2}{2} = \frac{3}{2} = 3 \cdot \frac{1}{2} = (m+1) \cdot \frac{1}{2} \\
h(3,1) &= \frac{1+2+3}{3} = \frac{6}{3} = 4 \cdot \frac{1}{2} = (m+1) \cdot \frac{1}{2} \\
h(3,3) &= 3 = 4 \cdot \frac{3}{4} = (m+1) \cdot \frac{3}{4} \\
h(4,3) &= \frac{\max(1,2,3) + \max(1,2,4) + \max(1,3,4) + \max(2,3,4)}{4} \\
&= \frac{3+4+4+4}{4} = \frac{15}{4} = 5 \cdot \frac{3}{4} = (m+1) \cdot \frac{3}{4}
\end{aligned} \tag{10}
$$

So maybe the general formula for $h(m,v)$ is:

$$
\boxed{h(m,v) = (m+1) \cdot \frac{v}{v+1}} \tag{11}
$$

2.4 A general formula for h(m,v)

We've found a potential formula for $h(m,v)$, but how can we show that the formula is true for all values of m and v?

Let's look at how we calculate $h(m,v)$.

Consider $h(5,3)$. The possible groups of 3 meals picked from 5 are:

- $\{5,4,3\}$ $\{5,4,2\}$ $\{5,4,1\}$ $\{5,3,2\}$ $\{5,3,1\}$ $\{5,2,1\}$
- $\{4,3,2\}$ $\{4,3,1\}$ $\{4,2,1\}$
- $\{3,2,1\}$

There are five 5's, three 4's, and one 3 representing the max score. Think of how we arrive at these numbers. For the 5, we pick all the combinations of 2 numbers from 1-4. For the 4, we pick all the combinations of 2 numbers from 1-3 (we don't pick from 5, since we accounted for all of the 5's first). And similarly for 3, we pick all the combinations of 2 numbers from 1-2.

The total number of combinations of meals is $\binom{5}{3}$. Therefore we can calculate $h(5,3)$ as:

$$
\begin{aligned}
h(5,3) &= \frac{\binom{4}{2} \cdot 5 + \binom{3}{2} \cdot 4 + \binom{2}{2} \cdot 3}{\binom{5}{3}} \\
&= \frac{6 \cdot 5 + 3 \cdot 4 + 1 \cdot 3}{10} = \frac{45}{10} = \frac{9}{2} = 6 \cdot \frac{3}{4}
\end{aligned} \tag{12}
$$

In addition to agreeing with our hypothesized formula from the previous section $h(m, v) = (m + 1) \cdot \frac{v}{v+1}$, we can now see an alternate formula for $h(m, v)$:

$$\boxed{h(m, v) = \frac{\sum_{i=v}^{m} \binom{i-1}{v-1} \cdot i}{\binom{m}{v}}} \tag{13}$$

This is the general formula for $h(m, v)$. Now we want to prove that this formula is the same as our simpler formula.

2.5 Proving the Correctness h(m,v)

We want to show that

$$h(m, v) = \frac{\sum_{i=v}^{m} \binom{i-1}{v-1} \cdot i}{\binom{m}{v}} = (m + 1) \cdot \frac{v}{v + 1} \tag{14}$$

First we'll show that (eq 14) is true for $v = m$. This will be the base case for a proof by induction.

$$\begin{aligned} h(m, m) &= \frac{\sum_{i=m}^{m} \binom{m-1}{m-1} \cdot i}{\binom{m}{m}} \\ &= \frac{\sum_{i=m}^{m} 1 \cdot i}{1} \\ &= m \\ &= (m + 1) \cdot \frac{m}{m + 1} \end{aligned} \tag{15}$$

Now we'll show that if (eq 14) is true for m, then it's true for $m + 1$.

For convenience, let's define

$$h^*(m, v) = \binom{m}{v} h(m, v) \tag{16}$$

We want to show that

$$h^*(m, v) = \binom{m}{v} \sum_{i=v}^{m} \binom{i-1}{v-1} \cdot i = \binom{m}{v} (m + 1) \cdot \frac{v}{v + 1} \tag{17}$$

Assume (eq 17) is true for m. Now let's look at $h^*(m + 1, v)$:

$$
\begin{aligned}
h^*(m+1,v) &= \sum_{i=v}^{m+1} \binom{i-1}{v-1} \cdot i \\
&= \Big[\sum_{i=v}^{m} \binom{i-1}{v-1} \cdot i\Big] + \binom{m+1-1}{v-1} \cdot (m+1) \\
&\textbf{substituting the assumption (eq 17) for the sum on the left} \\
&= \binom{m}{v}(m+1) \cdot \frac{v}{v+1} + \binom{m}{v-1} \cdot (m+1) \\
&\textbf{factoring out the } (m+1) \\
&= (m+1)\Big[\binom{m}{v} \cdot \frac{v}{v+1} + \binom{m}{v-1}\Big] \\
&= (m+1)\Big[\binom{m}{v} \cdot \frac{v}{v+1} + \binom{m}{v-1} \cdot \frac{v+1}{v+1}\Big] \\
&= (m+1)\Big[\frac{\binom{m}{v}v + \binom{m}{v-1}(v+1)}{v+1}\Big] \\
&= (m+1)\Big[\frac{v[\binom{m}{v} + \binom{m}{v-1}] + \binom{m}{v-1}}{v+1}\Big]
\end{aligned}
\tag{18}
$$

At this point, let's pause for a second to prove a well known fact about binomial coefficients.

$$
\begin{aligned}
&\binom{m}{v} + \binom{m}{v-1} \\
=&\frac{m!}{v!(m-v)!} + \frac{m!}{(v-1)!(m-v+1)!} \\
=&\frac{(m-v+1)m!}{v!(m-v+1)!} + \frac{vm!}{v!(m-v+1)!} \\
=&\frac{m!(m-v+1+v)}{v!(m-v+1)!} \\
=&\frac{m!(m+1)}{v!(m-v+1)!} \\
=&\frac{(m+1)!}{v!(m+1-v)!} \\
=&\binom{m+1}{v}
\end{aligned}
$$

So we see that

$$
\boxed{\binom{m}{v} + \binom{m}{v-1} = \binom{m+1}{v}} \tag{19}
$$

This is known as Pascal's Triangle. You can calculate any binomial coefficient by adding up the two coefficients above it in the triangle.

$m=0$							1						
$m=1$						1		1					
$m=2$					1		2		1				
$m=3$				1		3		3		1			
$m=4$			1		4		6		4		1		
$m=5$		1		5		10		10		5		1	
$m=6$	1		6		15		20		15		6		1

Using this fact, let's continue where we left off in (eq 18):

$$\begin{aligned} h^*(m+1,v) &= \sum_{i=v}^{m+1} \binom{i-1}{v-1} \cdot i \\ &= (m+1)\Big[\frac{v[\binom{m}{v} + \binom{m}{v-1}] + \binom{m}{v-1}}{v+1}\Big] \qquad (20) \\ & \textbf{using Pascal's Triangle (eq 19)} \\ &= (m+1)\Big[\frac{v\binom{m+1}{v} + \binom{m}{v-1}}{v+1}\Big] \end{aligned}$$

Let's solve for the top part of the above fraction.

$$\begin{aligned} & v\binom{m+1}{v} + \binom{m}{v-1} \\ =& v \cdot \frac{(m+1)!}{v!(m+1-v)!} + \frac{m!}{(v-1)!(m-(v-1))!} \\ =& \frac{v(m+1)!}{v!(m+1-v)!} + \frac{m!}{(v-1)!(m+1-v)!} \\ =& \frac{(m+1)!}{(v-1)!(m+1-v)!} + \frac{m!}{(v-1)!(m+1-v)!} \qquad (21) \\ =& \frac{(m+1)!+m!}{(v-1)!(m+1-v)!} \\ =& \frac{m![(m+1)+1]}{(v-1)!(m+1-v)!} \\ =& \frac{m!(m+2)}{(v-1)!(m+1-v)!} \end{aligned}$$

Substituting this back into (eq 20) gives:

$$\begin{aligned}
h^*(m+1,v) &= (m+1)\Big[\frac{v\binom{m+1}{v}+\binom{m}{v-1}}{v+1}\Big] \\
&= \frac{m+1}{v+1}\frac{m!(m+2)}{(v-1)!(m+1-v)!} \\
&= \frac{(m+2)(m+1)!}{(v+1)(v-1)!(m+1-v)!} \\
&\textbf{multiply by } \frac{v}{v} \\
&= \frac{v(m+2)(m+1)!}{v(v+1)(v-1)!(m+1-v)!} \\
&\textbf{combine } v \textbf{ with } (v-1)! \\
&= \frac{v(m+2)(m+1)!}{(v+1)v!(m+1-v)!} \\
&= (m+2)\cdot\frac{v}{v+1}\cdot\frac{(m+1)!}{v!(m+1-v)!} \\
&= (m+2)\cdot\frac{v}{v+1}\cdot\binom{m+1}{v}
\end{aligned} \tag{22}$$

So given

$$h^*(m,v) = \binom{m}{v}(m+1)\cdot\frac{v}{v+1} \tag{23}$$

We see that

$$h^*(m+1,v) = \binom{m+1}{v}(m+2)\cdot\frac{v}{v+1} \tag{24}$$

So by induction it's true for all m. Therefore we see that

$$\boxed{h(m,v) = \frac{h^*(m,v)}{\binom{m}{v}} = (m+1)\cdot\frac{v}{v+1}} \tag{25}$$

Note that proofs by induction require establishing that the formula holds for a base case. We did this earlier when we showed that the formula always holds when $m = v$. Since it must be that $m \geq v$ (there must be at least m items on the menu in order to try $v = m$ items), this covers our base case for all v's. Therefore the formula holds for all legal values of m and v.

2.6 Maximizing $f(m,t,v)$

We've found formulas for $g(m,v)$ and $h(m,v)$

$$\boxed{g(m,v) = \frac{m+1}{2}} \tag{26}$$

$$\boxed{h(m,v) = (m+1) \cdot \frac{v}{v+1}} \tag{27}$$

Now recall the formula for $f(m,t,v)$

$$\boxed{f(m,t,v) = v \cdot g(m,v) + (t-v) \cdot h(m,v)} \tag{28}$$

Substituting g and h into this formula we find

$$\begin{aligned} f(m,t,v) &= v \cdot \frac{m+1}{2} + (t-v)(m+1)\frac{v}{v+1} \\ &= (m+1)\left[\frac{v}{2} + (t-v)\frac{v}{v+1}\right] \end{aligned} \tag{29}$$

Notice what's happening here. All of the v different meals we try are average - i.e. they're on average worth $\frac{m+1}{2}$ points. The $t-v$ best meals that we eat are all above average unless we only try one meal.

The more variety meals we eat, the better the expected best meal gets - from 50th percentile for trying one meal, to 67th percentile for trying two meals, to 75th percentile for trying three meals, etc... The tradeoff is that the more v meals we try, the better the best meal we find is, but also the more average scores we add to our total. So we want to find the value of v that strikes a balance between finding a good meal while not bringing our overall average down too much.

In order to figure out which integer value of v yields the highest value of $f(m,t,v)$ we can simply solve the following equation for v:

$$f(m,t,v) = f(m,t,v-1) \tag{30}$$

The peak value of f will occur somewhere between v and $v-1$, so by figuring out the value of v that solves the above equation, we know that the closest integer $\leq v$ will be the integer value which maximizes the function.

$$
\begin{aligned}
&f(m,t,v) = f(m,t,v-1) \\
&\Rightarrow (m+1)\Big[\frac{v}{2} + \frac{tv-v^2}{v+1}\Big] = (m+1)\Big[\frac{v-1}{2} + \frac{t(v-1)-(v-1)^2}{v}\Big] \\
&\Rightarrow \frac{v}{2} + \frac{tv-v^2}{v+1} = \frac{v-1}{2} + \frac{t(v-1)-(v-1)^2}{v} \\
&\Rightarrow \frac{v}{2} - \frac{v-1}{2} + \frac{tv-v^2}{v+1} = \frac{tv-t-(v^2-2v+1)}{v} \\
&\Rightarrow \frac{1}{2} + \frac{tv-v^2}{v+1} - \frac{tv-t-v^2+2v-1}{v} = 0 \\
&\Rightarrow \frac{1}{2} + \frac{tv-v^2}{v+1} + \frac{v^2-2v+1-tv+t}{v} = 0 \\
&\Rightarrow \frac{1}{2} + \frac{tv^2-v^3}{v(v+1)} + \frac{(v+1)(v^2-2v+1-tv+t)}{v(v+1)} = 0 \\
&\Rightarrow \frac{v(v+1)}{2v(v+1)} + \frac{2tv^2-2v^3}{2v(v+1)} + \frac{2(v+1)(v^2-2v+1-tv+t)}{2v(v+1)} = 0 \\
&\Rightarrow v(v+1) + 2tv^2 - 2v^3 + 2(v+1)(v^2-2v+1-tv+t) = 0 \\
&\Rightarrow v^2 + v + 2tv^2 - 2v^3 + 2[(v^3-2v^2+v-tv^2+tv) + (v^2-2v+1-tv+t)] = 0 \\
&\Rightarrow v^2 + v + 2tv^2 - 2v^3 + 2v^3 - 4v^2 + 2v - 2tv^2 + 2tv + 2v^2 - 4v + 2 - 2tv + 2t = 0 \\
&\Rightarrow (v^2 - 4v^2 + 2v^2) + (v + 2v - 4v) + (2tv^2 - 2tv^2) + (2v^3 - 2v^3) + (2tv - 2tv) + 2 + 2t = 0 \\
&\Rightarrow -v^2 - v + 2 + 2t = 0 \\
&\Rightarrow v^2 + v - 2(t+1) = 0 \\
&\Rightarrow v = \frac{-1 + \sqrt{1 + 8(t+1)}}{2}
\end{aligned}
$$

So we find that the integer value of v is

$$
\boxed{v = \Big\lfloor \frac{\sqrt{8t+9}-1}{2} \Big\rfloor} \tag{31}
$$

But what if $v > m$? In this case, eating m meals will maximize the score. This is because the $f(m,t,v)$ function increases as v increases until v reaches $\Big\lfloor \frac{\sqrt{8t+9}-1}{2} \Big\rfloor$.